GHENT UNIVERSITY
GLOBAL CAMPUS

Mathematics

Topics you should know:

A. Calculus
    A. Basic properties (such as increasing/decreasing, positive/negative, zeros) and graphs of polynomial functions, rational functions, irrational (root) functions
    B. Absolute value function
    C. One-to-one functions and their inverses
    D. Basic properties (such as increasing/decreasing, positive/negative, zeros) and graphs of exponential and logarithmic functions
    E. Limits and continuity
    F. Vertical, horizontal and oblique asymptotes
    G. Definition of a derivative, intuitive definition of a derivative derivatives of basic functions, chain rule
    H. Applications of derivatives: extreme values, concave upward/downward, l'Hospital rule
    I. Equation of the tangent line to the curve of a graph at a certain point
    J. Definition of an antiderivative, antiderivatives of basic functions
    K. Connection between derivatives and antiderivatives
    L. Definite integrals
    M. Techniques of integration: substitution rule, integration by parts, partial fractions
B. Algebra
    A. Division of polynomials
    B. Binomium of Newton
    C. Solving of equations and inequalities involving polynomial, rational, irrational, exponential, logarithmic functions
    D. Solving of systems of equations
C. Trigonometry
    A. Degrees and radians
    B. Trigonometric functions sine, cosine, tangent, cotangent and their graphs
    C. Fundamental identity, addition and subtraction formulas, double-angle formulas, half-angle formulas, product-to-sum formulas, sum-to-product formulas
    D. Proving of trigonometric identities
    E. Trigonometric equations
    F. Trigonometry of right triangles
D. Geometry
    A. Points, coordinates and equations
    B. Equation of a circle
    C. Equation of a line
    D. Parallel and perpendicular lines

Two practice questions:

1. Consider the polynomial $P(x) = 4x^4 - 7x^3 + ax^2 + bx + 20$ with a and b real numbers such that $x - 1$ and $x + 2$ are factors of $P(x)$. Evaluate $P'(0)$.
   a. -29
   b. 12
   c. 17
   d. 29
   e. 34

Answer: b

The derivative of P is the function $P'(x) = 16\,x^3 - 21\,x^2 + 2ax + b$. Hence $P'(0) = b$. Since $x - 1$ is a factor of P, it holds that $P(1) = 0$ and thus that $4 - 7 + a + b + 20 = 0$. Similar, $P(-2) = 0$ implies that $64 + 56 + 4a - 2b + 20 = 0$. To find the value of b and thus of $P'(0)$ we need to solve the following system of equations.

$$\begin{cases} a + b + 17 = 0\ (1) \\ 2a - b + 70 = 0\ (2) \end{cases}$$

From (2) we obtain $b = 2a + 70$. Substituting in (1) gives $3a = -87$ or $a = -29$. It follows that $b = 2(-29) + 70 = 12$.

2. Consider the function $f: \mathbb{R} \longrightarrow \mathbb{R}: x \mapsto x - \sqrt{x^2 + 5x}$. Which of the following statements is true?
   a. The graph of f has a horizontal asymptote when x tends to $+\infty$ and an oblique asymptote when x tends to $-\infty$.
   b. The graph of f has a horizontal asymptote when x tends to $-\infty$ and an oblique asymptote when x tends to $+\infty$.
   c. The graph of f has a horizontal asymptote both when x tends to $+\infty$ and when x tends to $-\infty$.
   d. The graph of f has an oblique asymptote both when x tends to $+\infty$ and when x tends to $-\infty$.
   e. The graph of f has no asymptotes.

Answer: a

The function has a horizontal asymptote $y = -\frac{5}{2}$ when x tends to $+\infty$. Indeed,

$$\lim_{x \to +\infty} x - \sqrt{x^2 + 5x} = \lim_{x \to +\infty} \frac{\left(x - \sqrt{x^2 + 5x}\right)\left(x + \sqrt{x^2 + 5x}\right)}{x + \sqrt{x^2 + 5x}} = \lim_{x \to +\infty} \frac{-5x}{x + \sqrt{x^2 + 5x}}$$
$$= \lim_{x \to +\infty} \frac{-5}{1 + \sqrt{1 + {}^{5}/_{x}}} = \frac{-5}{2}$$

The function has an oblique asymptote $y = 2x + \frac{5}{2}$ when x tends to $-\infty$. Indeed,

$$\lim_{x \to -\infty} x - \sqrt{x^2 + 5x} - 2x = \lim_{x \to -\infty} -x - \sqrt{x^2 + 5x} = \lim_{x \to -\infty} \frac{(-x - \sqrt{x^2 + 5x})(-x + \sqrt{x^2 + 5x})}{-x + \sqrt{x^2 + 5x}}$$

$$= \lim_{x \to -\infty} \frac{-5x}{-x + \sqrt{x^2 + 5x}} = \lim_{x \to -\infty} \frac{-5}{-1 - \sqrt{1 + 5/x}} = \frac{5}{2}$$

Chemistry

Topics you should know:

- Elements, mixtures
- Lavoisier Law
- Symbolic representation of atoms and molecules, atomic mass, unit of atomic mass, electrons and nucleons (protons and neutrons)
- Oxidation number, ion and ion charge
- Reactions between bases and acids
- Reaction equations: ion exchange reactions, precipitation reactions, combustion reactions, synthesis reactions
- pH calculations, titration and titration reactions
- Bohr atom model, Bohr-Sommerfeld model, electron spin, Pauli rule
- Energy levels: s, p, d, f and orbital (basic knowledge)
- Electro negativity, electron pairs
- Covalent and ionic bonds, metal bonds
- Lewis notation from binary compounds and polyatomic compounds
- Polar and apolar compounds
- Intermolecular forces
- Nomenclature inorganic and organic compounds and ions (basic level)
- Stoichiometry: molar mass, molar volume, Avogadro constant, ideal gas law, mass density
- Concentration and concentration units, calculation of masses, volumes, concentrations, excess and limiting reagentia
- Reaction rate: factors influencing reaction rate, explanation via collision theory model
- Chemical equilibrium: equilibrium constant, factors influencing chemical equilibrium, calculations with equilibrium data
- Redox reactions: completion of redox reactions and interpretation of oxidators and reductors Sigma and pi bonds
- Solubility of ionic compounds

1. The fuel in the buster rockets of the Space Shuttle is constituted of a mixture of ammonium perchlorate, NH_4ClO_4, and aluminum powder. One of the reactions taking place during taking off is given by

$$6\ NH_4ClO_4(s) + 10\ Al(s) \longrightarrow 5\ Al_2O_3(s) + 3\ N_2(g) + 6\ HCl(g) + 9\ H_2O(g)$$

The indication (s) and (g) signify solid and gaseous phase. Suppose that these rockets are loaded with 11.75 tons of ammonium perchlorate and 2.70 tons of aluminum. Calculate the volume of nitrogen gas produced during taking off under normal conditions (0°C and atmospheric pressure, so that 1 mol of gas corresponds to 22.4 L)

(A) $6.72\ 10^2$ L
(B) $1.12\ 10^3$ L
(C) $7.47\ 10^4$ L
(D) $6.72\ 10^5$ L
(E) $1.12\ 10^6$ L

2. If a small zinc rod is put into a solution of tin(II)chloride, $SnCl_2$, tin will precipitate on the zinc rod and zinc will go into solution as Zn^{2+} ions.

$$Zn + SnCl_2 \longrightarrow Sn + ZnCl_2$$

If a lead rod is put into the same tin(II)chloride solution, no precipitation is observed.

$$Zn + SnCl_2 \longrightarrow Sn + ZnCl_2$$

What is your conclusion?

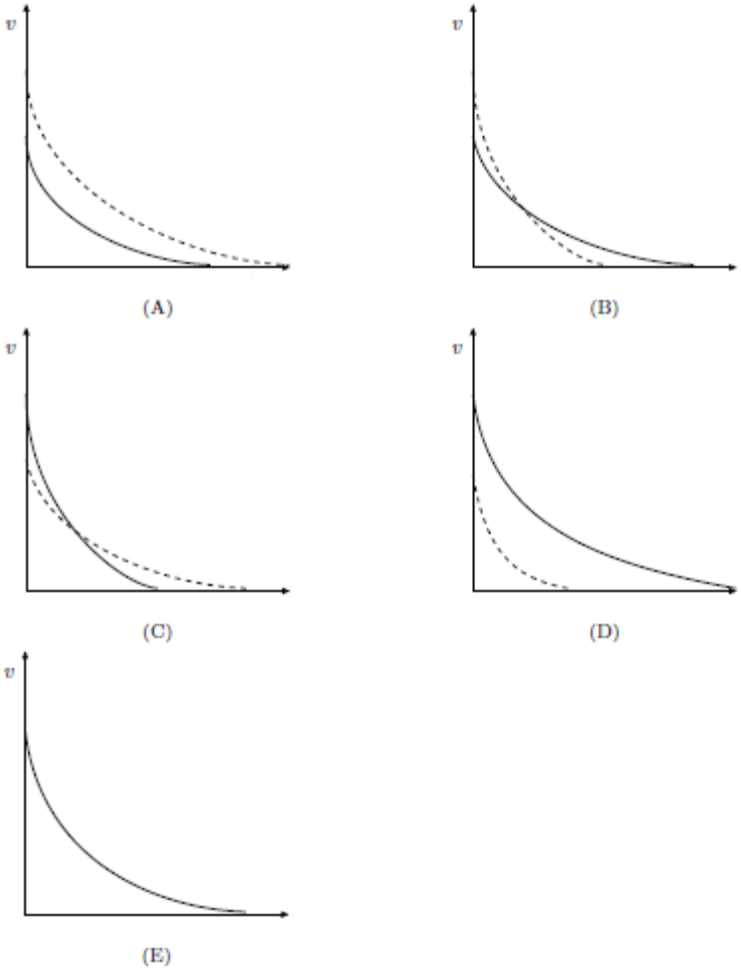
(A) Lead is a stronger reductor than zinc
(B) Tin is a stronger reductor than zinc
(C) Zinc is a stronger reductor than lead
(D) Tin is a stronger oxidator than zinc
(E) Zn^{2+} ions are a stronger oxidator than Pb^{2+} ions

3. Hydrogen peroxide, H_2O_2, spontaneously decomposes in water and oxygen gas

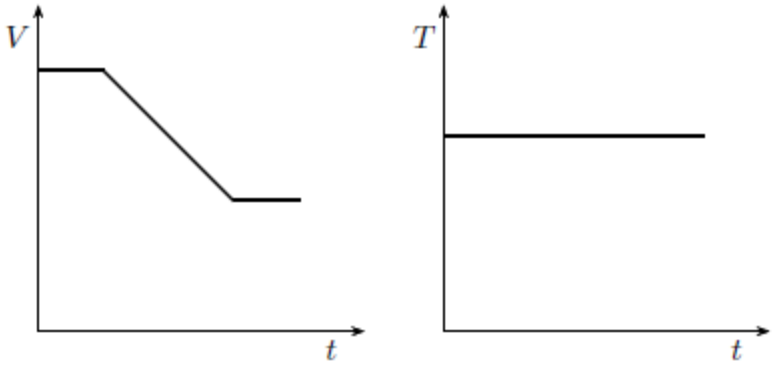
$$2\ H_2O_2 \longrightarrow 2\ H_2O + O_2$$

The reaction is one time performed without catalyst (full line) and one time in the presence of a catalyst (dashed line), starting from the same initial amount of hydrogen peroxide.

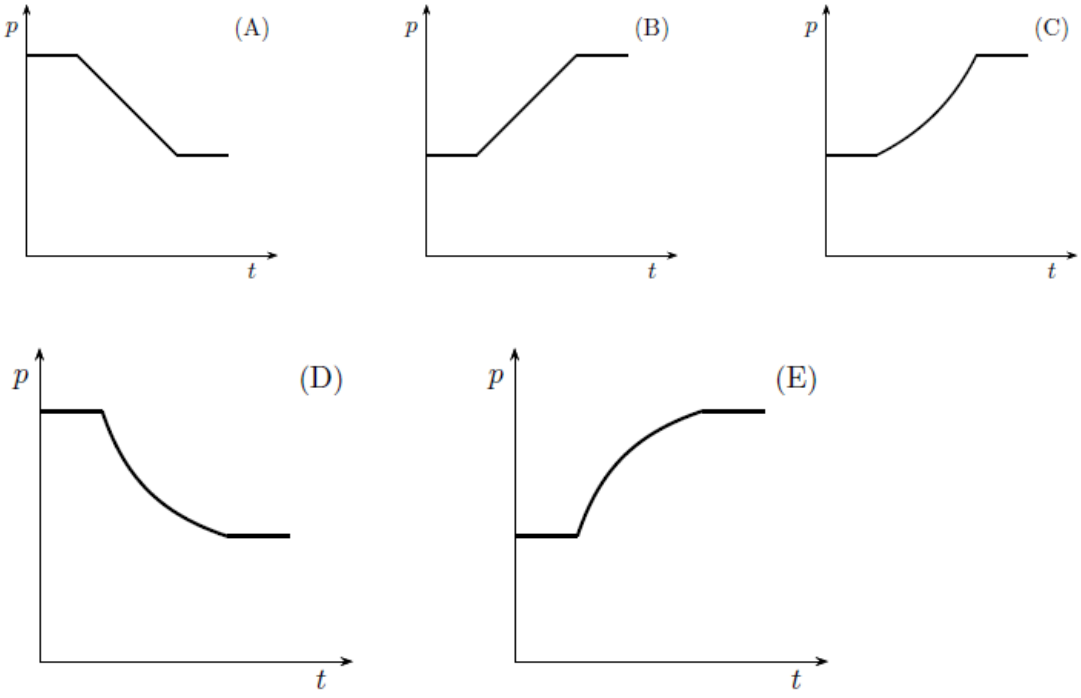
Which of the diagrams gives the correct reaction rate (v) as function of time (horizontal axis)?



4. A cylinder with movable piston is filled with an ideal gas. Hence, the relation pV = nRT holds (n is the number of moles). Next figures show the relation of the volume (V) and the temperature (T) as function of time.



Which graph represents the correct pressure p as function of time?



5. Sulfuric acid has a molar mass of 98 g/mol. In the laboratory there is 100 mL of a 0.10 M sulfuric acid solution. How much water needs to be added to this volume to prepare a solution containing 4.9 g/L of sulfuric acid?
   (A) 10 mL
   (B) 50 mL
   (C) 100 mL

(D) 150 mL
(E) 200 mL

6. The ion-electron equations for a redox reaction are given by

$$2\,I^- \rightarrow I_2(aq) + 2\,e^-$$

$$MnO_4^- + 8\,H^+ + 5\,e^- \rightarrow Mn^{2+}(aq) + 4\,H_2O$$

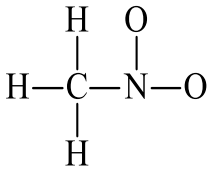
How many moles of iodide ions are oxidised by one mole of permanganate ions?

(A) 0.2
(B) 0.4
(C) 1
(D) 2
(E) 5

7. If the atomic number of an atom is represented by X and the mass number is represented by Y, the number of neutrons in an atom of this element is
(A) Y – X
(B) X – Y
(C) X + Y
(D) Y

8. Which of the following is the correct arrangement of electrons in a phosphorus atom (atomic number 15) ?
(A) 2, 8, 4, 1
(B) 2, 8, 5
(C) 5, 8, 2
(D) 2, 5, 8
(E) 2, 4, 8, 1

9. In the CH_3NO_2 molecule, shown below, what hybrid orbital set is used by the nitrogen atom for bonding? Be sure to check the Lewis structure to make sure it is correct?



(A) sp
(B) sp^2
(C) sp^3
(D) sp^3d
(E) sp^3d^2

10. Which of the following temperatures has an equal value expressed in °C and °F?
(A) -40
(B) -22.2
(C) 0
(D) 22.2
(E) +40

11. Consider the function $f: \mathbb{R} \to \mathbb{R}: x \mapsto x\cos(x^2)$. Evaluate the derivative of f at the point $\sqrt{2\pi}/2$.
a. $f'\left(\sqrt{2\pi}/2\right) = -\pi$
b. $f'\left(\sqrt{2\pi}/2\right) = -\sqrt{2\pi}$
c. $f'\left(\sqrt{2\pi}/2\right) = \sqrt{2\,\pi}$
d. $f'\left(\sqrt{2\pi}/2\right) = 0$
e. $f'\left(\sqrt{2\pi}/2\right) = 1 - \sqrt{2\pi}$

12. Consider a natural number $m \neq 0$. Evaluate $\lim_{n \to \infty} \frac{nm}{m-n}$.

a. $\frac{m}{m-1}$
b. m
c. 1
d. -1
e. $-m$

13. Consider the circle with equation $y^2 - 2y + x^2 + 6x - 15 = 0$. If $M = (a, b)$ is its center and R its radius, then $2a + b + R^2$ is equal to
    a. 10
    b. 14
    c. 20
    d. 24
    e. 30

14. Evaluate $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$.
    a. $e^{\sqrt{x}}$ + constant
    b. $2e^{\sqrt{x}}$ + constant
    c. $e^{-\sqrt{x}}$ + constant
    d. $\sqrt{x}e^{\sqrt{x}}$ + constant
    e. $\frac{1}{2}e^{\sqrt{x}}$ + constant

15. The trigonometric expression $\frac{1}{\cos x \tan x} - \sin x$ equals
    a. $\frac{1}{\sin x}$
    b. $\frac{\sin x}{\tan x}$
    c. $\frac{(\sin x)^2}{\cos x}$
    d. $\frac{\cos x}{\sin x}$
    e. $\frac{(\cos x)^2}{\sin x}$

16. A function $f: A \to B: x \mapsto f(x)$ is a one-to-one function is for all $x, y \in A$ it holds that: if $x \neq y$, then $f(x) \neq f(y)$. Determine which of the following functions is one-to-one.
    a. $f: \mathbb{N} \times \mathbb{N} \to \mathbb{N}: (n, m) \mapsto m + n$
    b. $f: \mathbb{N} \times \mathbb{N} \to \mathbb{N}: (n, m) \mapsto mn$
    c. $f: \mathbb{N} \times \mathbb{N} \to \mathbb{N}: (n, m) \mapsto 3^m 5^n$
    d. $f: \mathbb{N} \times \mathbb{N} \to \mathbb{N}: (n, m) \mapsto m^n$

e. $f: \mathbb{N} \times \mathbb{N} \to \mathbb{N}: (n, m) \mapsto 2^{m+n}$

17. Consider the function $f: \mathbb{R} \longrightarrow \mathbb{R}: x \mapsto x - \sqrt{x^2 + 5x}$. Which of the following statements is true?
    a. f is increasing on its domain.
    b. f is decreasing on its domain.
    c. f has two distinct zeros.
    d. $f(x) \leq 0$ for all x in its domain.
    e. f has both negative as positive values on its domain

18. Suppose a, b, c and d are real numbers such that $ab < cd$ and $0 < a < c$. Which of the following statements is definitely true?
    a. $a < cd$
    b. $b < d$
    c. $a < \frac{cd}{b}$
    d. If $d < 0$, then $b < 0$.
    e. If $b < 0$, then $d < 0$.

19. The function sgn is defined as follows: $sgn(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Evaluate $\int_0^4 x\, sgn(2 - x)\, dx$.
    a. 8
    b. 4
    c. 0
    d. -4
    e. -8

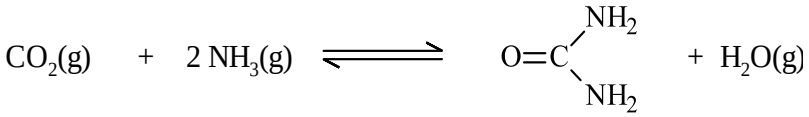
20. Suppose $f: \mathbb{R} \longrightarrow \mathbb{R}$ is a differentiable function such that $f(x) \geq 0$ for all $x \in \mathbb{R}$ and $P(5,9)$ is a point on the graph of f. The tangent line to the curve of f at P intersects the x-axis at the point $Q(1,0)$. If a function h is defined as $h: \mathbb{R} \longrightarrow \mathbb{R}: x \mapsto \sqrt{f(x)}$, then $h'(5)$ equals:
    a. $\frac{3}{8}$
    b. $\frac{3}{2}$
    c. $\frac{1}{6}$
    d. $\frac{9}{8\sqrt{5}}$
    e. $\frac{2}{27}$

Correct answers:

1) D
2) C
3) B
4) C
5) C
6) E
7) A
8) B
9) B
10) A
11) A
12) E
13) C
14) B
15) E
16) C
17) D
18) D
19) D
20) A

Two practice questions:

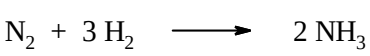
21. The German chemist Friedrich Wöhler discovered in 1828 that the carbon compound urea, $(NH_2)_2CO$, could be prepared from anorganic compounds. This discovery was very important for chemistry because it showed that natural compounds could be prepared in the chemical laboratory without using biological reagents. A possible synthesis of urea is



CO_2, NH_3 and H_2O are gaseous, urea is solid and the reaction is exothermic.
How can the temperature and pressure be adjusted so that the equilibrium shifts towards the side of urea?

(A) Increase temperature and decrease pressure
(B) Increase temperature and pressure
(C) Decrease temperature and pressure
(D) Decrease temperature and increase pressure
(E) Decrease temperature, a change in pressure does not affect the equilibrium

22. Ammonia is prepared according to the reaction



The reaction starts with a total pressure of 100 bar, nitrogen and hydrogen together, in a confined space at constant temperature. Initially the pressure of nitrogen is 62.5 bar and the pressure of hydrogen is 37.5 bar. What is the total pressure after completion of the reaction?

(A) 25 bar
(B) 50 bar
(C) 75 bar
(D) 100 bar
(E) 125 bar

Solution question 1

The reaction is exothermic, so Le Chatelier's principle can be adopted: 'When a system at equilibrium is subjected to change in concentration, temperature, volume, or pressure, then the system readjusts itself to (partially) counteract the effect of the applied change and a new equilibrium is established.' Hence, the temperature should be decreased.

Also, when there is an increase in pressure, the equilibrium will shift towards the side of the reaction with fewer moles of gas. Since urea is a solid, the reaction will proceed to the right in case of a pressure increase.

Final answer: a decrease in temperature and an increase in pressure will result in higher production of urea from carbon dioxide and ammonia.

Solution question 2

Total pressure is 100 bar, 62.5 bar for nitrogen and 37.5 bar for hydrogen. The stoichiometry of the reaction learns that 1 mole of nitrogen requires 3 moles of hydrogen. Since less hydrogen is present than nitrogen, the former is the so-called limiting reagent and will be consumed completely.

Hence, the hydrogen needs 37.5/3 = 12.5 bar of nitrogen, leaving 62.5 - 12.5 = 50 bar of nitrogen unused. In the given chemical reaction, 4 moles are converted into 2 moles so, via the ideal gas law (P*V = n*R*T), the pressure of 12.5 + 37.5 = 50 bar is reduced into 25 bar. Adding up gives a total pressure of 75 bar and the composition is 1/3 ammonia and 2/3 nitrogen.

Final answer: the total pressure, after completion of the reaction, is 75 bar.